On the Electromagnetic Field in a Cavity Fed by a Tangential Electric Field in an Aperture in its Wall

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***Abstract*—Heretofore, the electromagnetic field produced by a specified tangential electric field in an aperture in the wall of an arbitrarily shaped cavity has most often been expanded in terms of cavity modes. An alternative approach, that of the electric field integral equation is presented. In this approach, the cavity field is expressed as the field of a surface density of tangential electric current, or a surface density of tangential magnetic current, or a combination of surface densities of tangential electric and magnetic currents on the boundary of the cavity. Each surface density is characterized by a single tangential vector function which is determined by the integral equation requiring that the part of the electric field tangent to the boundary of the cavity must reduce to the specified tangential electric field in the aperture and zero elsewhere on the boundary of the cavity. The electric field integral equation method is specialized to more easily determine the field inside an arbitrary cylindrical cavity excited by a tangential electric field in an aperture in its lateral wall. The method is further specialized to a circular cavity.**

I. Introduction

CONSIDER an arbitrarily shaped cavity that is source-free and bounded by a closed surface S that is perfectly conducting everywhere except in an aperture. The boundary conditions require that the tangential electric field vanishes on the perfectly conducting part of S and is equal to a specified vector function in the aperture. The problem is to find the electromagnetic field in this cavity. It is of particular interest to find the tangential magnetic field in the aperture because this field is needed to carry out the generalized network formulation for aperture problems whereby each aperture that provides electromagnetic communication between regions such as cavities, waveguides, and half spaces is closed with an infinitely thin perfectly conducting plate, a magnetic current sheet $\boldsymbol{M}$ is placed on one side of the plate, $-\boldsymbol{M}$ is placed on the other side, and the tangential magnetic field on one side of the plate is set equal to that on the other side [1].

The usual method of solution is to express the electric field in the cavity as a linear combination of resonant electric fields, and to express the magnetic field as a linear combination of resonant magnetic fields. This method, called the modal expansion method, is advocated in [2, Ch. 5], [3, Ch. 3], and the references cited therein. The modal expansions of the electric and magnetic fields in a bounded region are concisely given by [4, (4a) and (4b)] and supporting equations [4, (5)–(17)]. Aside from two minor errors ("$\hat{n} \cdot \boldsymbol{e}_\nu = 0$" does not belong in [4, (6b)] and $\boldsymbol{e}_\nu$ should be replaced by $\boldsymbol{E}_\nu$ in [4, (9b)]), the modal expansion method is clearly described in [4]. Disadvantages of the modal expansion method will be pointed out in Section VI.

An alternative method called the EFIE (electric field integral equation) method is presented to obtain the field inside an arbitrarily shaped cavity due to a specified tangential electric field in an aperture. In this method, the cavity field is expressed as the field of a surface density of tangential electric current, or a surface density of tangential magnetic current, or a combination of surface densities of tangential electric and magnetic currents on the boundary of the cavity. The latter field is obviously source-free inside the cavity. The above surface density or combination of surface densities is characterized by a single tangential vector function which is determined by satisfying the boundary condition stated in the abstract. This condition gives an integral equation that is similar to the electric field integral equation encountered in the problem of scattering of an exterior field by a perfectly conducting closed surface. Variations of the EFIE method were used in [5] for the two-dimensional cavity region of a thick slit, in [6] for a small aperture in a cavity, and in [7, Section 8] for an aperture in one of the end faces of a cylindrical cavity. Although the finite element method [8] is well-suited for inhomogeneous regions and the modal expansion method [4] is appropriate for regions containing volume sources, the EFIE method is more suitable and efficient for homogeneous geometries where the excitation consists of a specified tangential electric field on the boundary rather than volume sources.

In Section II, the EFIE method is specialized to more easily determine the field inside an arbitrary cylindrical cavity excited by a specified tangential electric field in an aperture in its lateral wall. In Section III, the EFIE method is used to obtain a novel solution for the field inside a circular cylindrical cavity fed by a specified tangential electric field in an aperture in its lateral wall.

II. The Electric Field Integral Equation Method for a Cylindrical Cavity

The field in a cylindrical cavity excited by a specified tangential electric field in an aperture in one of its end faces was obtained in [7, Section 8]. In the rest of the present section,

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the EFIE method is specialized to more easily determine the field in a cylindrical cavity excited by a specified tangential electric field in an aperture in its lateral wall.

Consider the cylindrical cavity that extends from $z = 0$ to $z = L$ along its z-axis and whose wall contour in the transverse xy-plane is C. In its lateral wall, this cavity has an aperture in which the tangential electric field $\boldsymbol{E}_A$ is specified

$$\boldsymbol{E}_A = \hat{l}E_{Al}(l,z) + \hat{z}E_{Az}(l,z) \tag{1}$$

where $\hat{l}$ and $\hat{z}$ are the unit vectors in the l- and z-directions and l is a curvilinear coordinate measured along C. Of course, $E_{Al} = E_{Az} = 0$ on the wall, which is assumed to be perfectly conducting.

In the EFIE method, the electric and magnetic fields $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r})$ inside the cylindrical cavity are expanded as axial modes

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{p=0}^{P} \boldsymbol{E}_p^{\rm TM}(\boldsymbol{r}) + \sum_{p=1}^{P} \boldsymbol{E}_p^{\rm TE}(\boldsymbol{r}) \tag{2}$$

$$\boldsymbol{H}(\boldsymbol{r}) = \sum_{p=0}^{P} \boldsymbol{H}_p^{\rm TM}(\boldsymbol{r}) + \sum_{p=1}^{P} \boldsymbol{H}_p^{\rm TE}(\boldsymbol{r}) \tag{3}$$

where P is a sufficiently large integer. Also, $\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r})$ and $\boldsymbol{H}_p^{\rm TM}(\boldsymbol{r})$ are the electric and magnetic fields of the surface density $\hat{z}J_p(l)\cos(k_p z)$ of electric current on the associated waveguide wall. This wall is the lateral wall of the cavity extended to $-\infty$ in the negative z-direction and to $+\infty$ in the positive z-direction. In (2) and (3), $\boldsymbol{E}_p^{\rm TE}(\boldsymbol{r})$ and $\boldsymbol{H}_p^{\rm TE}(\boldsymbol{r})$ are the electric and magnetic fields of the surface density $\hat{z}M_p(l)\sin(k_p z)$ of magnetic current on the associated waveguide wall. We choose

$$k_p = \frac{p\pi}{L}, \quad p = 0, 1, 2, \ldots, P \tag{4}$$

so that the tangential components of $\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r})$ and $\boldsymbol{E}_p^{\rm TE}(\boldsymbol{r})$ will vanish at $z = 0$ and $z = L$ (look ahead at (13) and (14)). By using extended lateral wall currents that go from $z = -\infty$ to $z = \infty$, end face currents are no longer required.

The fields $\boldsymbol{H}_p^{\rm TM}(\boldsymbol{r})$, $\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r})$, $\boldsymbol{E}_p^{\rm TE}(\boldsymbol{r})$, and $\boldsymbol{H}_p^{\rm TE}(\boldsymbol{r})$ are given by [9, Section (3–12)]

$$\boldsymbol{H}_p^{\rm TM}(\boldsymbol{r}) = \nabla \times \left(\hat{z}\psi_p^{\rm TM}\cos(k_p z)\right) \tag{5}$$

$$\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r}) = \frac{1}{j\omega\epsilon}\nabla \times \boldsymbol{H}_p^{\rm TM}(\boldsymbol{r}) \tag{6}$$

$$\boldsymbol{E}_p^{\rm TE}(\boldsymbol{r}) = -\nabla \times \left(\hat{z}\psi_p^{\rm TE}\sin(k_p z)\right) \tag{7}$$

$$\boldsymbol{H}_p^{\rm TE}(\boldsymbol{r}) = \frac{-1}{j\omega\mu}\nabla \times \boldsymbol{E}_p^{\rm TE}(\boldsymbol{r}) \tag{8}$$

where μ is the permeability, ϵ is the permittivity, and $e^{j\omega t}$ time dependence is assumed. The quantities $\psi_p^{\rm TM}$ and $\psi_p^{\rm TE}$ are given as functions of curvilinear coordinates n and l by

$$\psi_p^{\rm TM}(n,l) = \frac{1}{4j}\int_C J_p(l')H_0^{(2)}(k_\rho|\boldsymbol{\rho} - \boldsymbol{\rho}'|)h_l(n_0, l')dl' \tag{9}$$

$$\psi_p^{\rm TE}(n,l) = \frac{1}{4j}\int_C M_p(l')H_0^{(2)}(k_\rho|\boldsymbol{\rho} - \boldsymbol{\rho}'|)h_l(n_0, l')dl'. \tag{10}$$

Here, n is such that (n, l, z) form a right-handed orthogonal curvilinear coordinate system. On C, the n-direction is outward from the cavity and $n = n_0$ where n_0 is constant over all C. In (9) and (10), $H_0^{(2)}$ is the Hankel function of the second kind of order zero, $\boldsymbol{\rho}$ is the radius vector from the origin in the transverse plane to the point whose curvilinear coordinates are (n, l) in the transverse plane, $\boldsymbol{\rho}'$ is the transverse radius vector from the origin to the point on C whose curvilinear coordinates are (n_0, l'), and h_l is the metric coefficient [10, (162) on p. 496] associated with l. Moreover,

$$k_\rho = \sqrt{k^2 - k_p^2} \tag{11}$$

where $k = \omega\sqrt{\mu\epsilon}$.

The expressions of (5)–(8) in the curvilinear coordinate system (n, l, z) are [10, (166)]

$$\boldsymbol{H}_p^{\rm TM}(\boldsymbol{r}) = \left(\hat{n}\frac{1}{h_l}\frac{\partial\psi_p^{\rm TM}}{\partial l} - \hat{l}\frac{1}{h_n}\frac{\partial\psi_p^{\rm TM}}{\partial n}\right)\cos(k_p z) \tag{12}$$

$$\begin{aligned}\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r}) = {} & \frac{jk_p\eta}{k}\left[\hat{n}\frac{1}{h_n}\frac{\partial\psi_p^{\rm TM}}{\partial n} + \hat{l}\frac{1}{h_l}\frac{\partial\psi_p^{\rm TM}}{\partial l}\right]\sin(k_p z) \\ & + \hat{z}\frac{j\eta}{kh_n h_l}\left[\frac{\partial}{\partial n}\left(\frac{h_l}{h_n}\frac{\partial\psi_p^{\rm TM}}{\partial n}\right)\right. \\ & \left. + \frac{\partial}{\partial l}\left(\frac{h_n}{h_l}\frac{\partial\psi_p^{\rm TM}}{\partial l}\right)\right]\cos(k_p z)\end{aligned} \tag{13}$$

$$\boldsymbol{E}_p^{\rm TE}(\boldsymbol{r}) = \left(-\hat{n}\frac{1}{h_l}\frac{\partial\psi_p^{\rm TE}}{\partial l} + \hat{l}\frac{1}{h_n}\frac{\partial\psi_p^{\rm TE}}{\partial n}\right)\sin(k_p z) \tag{14}$$

$$\begin{aligned}\boldsymbol{H}_p^{\rm TE}(\boldsymbol{r}) = {} & \frac{-jk_p}{k\eta}\left[\hat{n}\frac{1}{h_n}\frac{\partial\psi_p^{\rm TE}}{\partial n} + \hat{l}\frac{1}{h_l}\frac{\partial\psi_p^{\rm TE}}{\partial l}\right]\cos(k_p z) \\ & + \hat{z}\frac{j}{k\eta h_n h_l}\left[\frac{\partial}{\partial n}\left(\frac{h_l}{h_n}\frac{\partial\psi_p^{\rm TE}}{\partial n}\right)\right. \\ & \left. + \frac{\partial}{\partial l}\left(\frac{h_n}{h_l}\frac{\partial\psi_p^{\rm TE}}{\partial l}\right)\right]\sin(k_p z)\end{aligned} \tag{15}$$

where $\eta = \sqrt{\mu/\epsilon}$ and h_n and h_l are the metric coefficients associated with n and l, respectively. Now, $\psi_p^{\rm TM}$ and $\psi_p^{\rm TE}$ satisfy

$$\nabla^2\psi_p^{\rm TM} + k_\rho^2\psi_p^{\rm TM} = 0 \tag{16}$$

$$\nabla^2\psi_p^{\rm TE} + k_\rho^2\psi_p^{\rm TE} = 0. \tag{17}$$

Equations (16) and (17) with ∇^2 given by [10, (167)] simplify the $\hat{z}$-components of $\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r})$ and $\boldsymbol{H}_p^{\rm TE}(\boldsymbol{r})$ of (13) and (15):

$$\begin{aligned}\boldsymbol{E}_p^{\rm TM}(\boldsymbol{r}) = {} & \frac{jk_p\eta}{k}\left[\hat{n}\frac{1}{h_n}\frac{\partial\psi_p^{\rm TM}}{\partial n} + \hat{l}\frac{1}{h_l}\frac{\partial\psi_p^{\rm TM}}{\partial l}\right]\sin(k_p z) \\ & - \hat{z}\frac{j\eta k_\rho^2\psi_p^{\rm TM}}{k}\cos(k_p z)\end{aligned} \tag{18}$$

$$\begin{aligned}\boldsymbol{H}_p^{\rm TE}(\boldsymbol{r}) = {} & \frac{-jk_p}{k\eta}\left[\hat{n}\frac{1}{h_n}\frac{\partial\psi_p^{\rm TE}}{\partial n} + \hat{l}\frac{1}{h_l}\frac{\partial\psi_p^{\rm TE}}{\partial l}\right]\cos(k_p z) \\ & - \hat{z}\frac{jk_\rho^2\psi_p^{\rm TE}}{k\eta}\sin(k_p z).\end{aligned} \tag{19}$$

Substituting (18) and (14) into (2), one has

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) = \hat{n}\sum_{p=1}^{P}\left(\frac{jk_p\eta}{kh_n}\frac{\partial\psi_p^{\mathrm{TM}}}{\partial n} - \frac{1}{h_l}\frac{\partial\psi_p^{\mathrm{TE}}}{\partial l}\right)\sin(k_pz) \\ + \hat{l}\sum_{p=1}^{P}\left(\frac{jk_p\eta}{kh_l}\frac{\partial\psi_p^{\mathrm{TM}}}{\partial l} + \frac{1}{h_n}\frac{\partial\psi_p^{\mathrm{TE}}}{\partial n}\right)\sin(k_pz) \\ - \hat{z}\left(\frac{j\eta}{k}\right)\sum_{p=0}^{P}k_\rho^2\psi_p^{\mathrm{TM}}\cos(k_pz). \qquad (20)\end{aligned}$$

A Fourier series expansion of (1) is

$$\boldsymbol{E}_A = \hat{l}\sum_{p=1}^{P}E_{Alp}(l)\sin(k_pz) + \hat{z}\sum_{p=0}^{P}E_{Azp}(l)\cos(k_pz) \quad (21)$$

where

$$E_{Alp}(l) = \frac{2}{L}\int_0^L E_{Al}(l,z)\sin(k_pz)\,dz \quad (22)$$

$$E_{Azp}(l) = \frac{\varepsilon_p}{L}\int_0^L E_{Az}(l,z)\cos(k_pz)\,dz \quad (23)$$

in which

$$\varepsilon_p = \begin{cases}1, & p=0\\ 2, & p=1,2,\ldots.\end{cases} \quad (24)$$

Requiring that the l- and z-components of (20) reduce to those of $\boldsymbol{E}_A$ of (21) on the lateral wall of the cavity where $n = n_0$, one has

$$\frac{1}{h_n(n_0,l)}\left[\frac{\partial\psi_p^{\mathrm{TE}}(n,l)}{\partial n}\right]_{n=n_0} + \frac{jk_p\eta}{kh_l(n_0,l)}\frac{\partial\psi_p^{\mathrm{TM}}(n_0,l)}{\partial l} = E_{Alp}(l) \quad (25)$$

$$-\frac{jk_\rho^2\eta\psi_p^{\mathrm{TM}}(n_0,l)}{k} = E_{Azp}(l). \quad (26)$$

Substitution of (9) into (26) gives the integral equation

$$-\frac{\eta k_\rho^2}{4k}\int_C J_p(l')H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\,h_l(n_0,l')\,dl' = E_{Azp}(l) \quad (27)$$

where $\boldsymbol{\rho}$ is now the transverse radius vector to the point (n_0, l). (27) is an integral equation that can be solved numerically for J_p. This integral equation is similar to the electric field integral equation encountered in the two-dimensional problem of TM scattering by a perfectly conducting cylinder.

As for the quantity $\left[\frac{\partial}{\partial n}\psi_p^{\mathrm{TE}}(n,l)\right]_{n=n_0}$ in (25), differentiating both sides of (10) with respect to n, letting $\frac{\partial}{\partial n}$ operate on $H_0^{(2)}$ rather than on the result of integration, and then letting n approach n_0 from inside the cavity, we see that

$$\left[\frac{\partial\psi_p^{\mathrm{TE}}(n,l)}{\partial n}\right]_{n=n_0} = \frac{1}{4j} \cdot \lim_{n\to n_0}\left\{\int_C M_p(l')\frac{\partial}{\partial n}H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\,h_l(n_0,l')\,dl'\right\}. \quad (28)$$

Substitution of (28) into (25) gives

$$\lim_{n\to n_0}\left\{\frac{1}{4jh_n(n_0,l)}\int_C M_p(l') \cdot\frac{\partial}{\partial n}H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\,h_l(n_0,l')\,dl'\right\} = E_{Alp}(l) - \frac{jk_p\eta}{kh_l(n_0,l)}\frac{\partial\psi_p^{\mathrm{TM}}(n_0,l)}{\partial l}. \quad (29)$$

The quantity $\frac{\partial}{\partial l}\psi_p^{\mathrm{TM}}(n_0,l)$ is obtained by substituting the solution $J_p(l')$ of (27) into (9) and then differentiating (9) with respect to l. Once this is done, the entire right-hand side of (29) is known. In the limit as n approaches n_0 from inside the cavity, (29) becomes [11, Section 3.1.2]

$$\frac{1}{2}M_p(l) + \frac{1}{4jh_n(n_0,l)}\int_C M_p(l')\left[\frac{\partial}{\partial n}H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\right]_{n=n_0} \cdot h_l(n_0,l')\,dl' = E_{Alp}(l) - \frac{jk_p\eta}{kh_l(n_0,l)}\frac{\partial\psi_p^{\mathrm{TM}}(n_0,l)}{\partial l}. \quad (30)$$

In contrast to that in (29), the integrand in (30) is not defined at $l' = l$. The integral in (30) must be evaluated by deleting a section of C containing the point l and passing to the limit as the length of this section approaches zero. Equations (29) and (30) are two different forms of the same integral equation. This integral equation is similar to the electric field integral equation encountered in the two-dimensional problem of TE scattering by a perfectly conducting cylinder when the scattered field is expressed as the field of z-directed magnetic current on the surface of the cylinder. The integral equations (27), (29), and (30) can be solved by a numerical procedure such as the method of moments [12].

The numerical solutions for J_p and M_p are substituted into expressions (9) and (10) for ψ_p^{TM} and ψ_p^{TE}. These ψ_p's determine $\boldsymbol{E}_p^{\mathrm{TM}}(\boldsymbol{r})$, $\boldsymbol{H}_p^{\mathrm{TM}}(\boldsymbol{r})$, $\boldsymbol{E}_p^{\mathrm{TE}}(\boldsymbol{r})$, and $\boldsymbol{H}_p^{\mathrm{TE}}(\boldsymbol{r})$ according to (18), (12), (14), and (19), respectively. Finally, $\boldsymbol{E}_p^{\mathrm{TM}}(\boldsymbol{r})$, $\boldsymbol{H}_p^{\mathrm{TM}}(\boldsymbol{r})$, $\boldsymbol{E}_p^{\mathrm{TE}}(\boldsymbol{r})$, and $\boldsymbol{H}_p^{\mathrm{TE}}(\boldsymbol{r})$ are substituted into expressions (2) and (3) for the electric and magnetic fields in the cavity.

III. The Electric Field Integral Equation Method for a Circular Cavity

Consider the special case where the cylindrical cavity in Section II is the circular cavity of radius a bounded by end faces at $z = 0$ and $z = L$ and the lateral wall at $\rho = a$. The usual cylindrical coordinates (ρ, ϕ, z) are used throughout Section III. Equation (1) specializes to

$$E_A = \hat{\phi}E_{A\phi}(\phi,z) + \hat{z}E_{Az}(\phi,z). \quad (31)$$

Equations (2)–(8) remain unchanged. However, $J_p(l)$ and $M_p(l)$ specialize to $J_p(\phi)$ and $M_p(\phi)$ so that (9) and (10) become

$$\psi_p^{\mathrm{TM}}(\rho,\phi) = \frac{a}{4j}\int_0^{2\pi}J_p(\phi')H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\,d\phi' \quad (32)$$

$$\psi_p^{\mathrm{TE}}(\rho,\phi) = \frac{a}{4j}\int_0^{2\pi}M_p(\phi')H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)d\phi' \quad (33)$$

where k_ρ is given by (11).

Equations (12), (18), (14), and (19) reduce to, respectively,

$$H_p^{\mathrm{TM}}(\boldsymbol{r}) = \left(\frac{\hat{\rho}}{\rho}\frac{\partial \psi_p^{\mathrm{TM}}}{\partial \phi} - \hat{\phi}\frac{\partial \psi_p^{\mathrm{TM}}}{\partial \rho}\right)\cos(k_p z) \quad (34)$$

$$E_p^{\mathrm{TM}}(\boldsymbol{r}) = \frac{jk_p\eta}{k}\left(\hat{\rho}\frac{\partial \psi_p^{\mathrm{TM}}}{\partial \rho} + \frac{\hat{\phi}}{\rho}\frac{\partial \psi_p^{\mathrm{TM}}}{\partial \phi}\right)\sin(k_p z) - \hat{z}\left(\frac{jk_\rho^2\eta\psi_p^{\mathrm{TM}}}{k}\right)\cos(k_p z) \quad (35)$$

$$E_p^{\mathrm{TE}}(\boldsymbol{r}) = \left(-\frac{\hat{\rho}}{\rho}\frac{\partial \psi_p^{\mathrm{TE}}}{\partial \phi} + \hat{\phi}\frac{\partial \psi_p^{\mathrm{TE}}}{\partial \rho}\right)\sin(k_p z) \quad (36)$$

$$H_p^{\mathrm{TE}}(\boldsymbol{r}) = \frac{-jk_p}{k\eta}\left(\hat{\rho}\frac{\partial \psi_p^{\mathrm{TE}}}{\partial \rho} + \frac{\hat{\phi}}{\rho}\frac{\partial \psi_p^{\mathrm{TE}}}{\partial \phi}\right)\cos(k_p z) - \hat{z}\left(\frac{jk_\rho^2\psi_p^{\mathrm{TE}}}{k\eta}\right)\sin(k_p z). \quad (37)$$

Substituting (35) and (36) into (2), one has

$$\boldsymbol{E}(\boldsymbol{r}) = \hat{\rho}\sum_{p=1}^{P}\left(\frac{jk_p\eta}{k}\frac{\partial \psi_p^{\mathrm{TM}}}{\partial \rho} - \frac{1}{\rho}\frac{\partial \psi_p^{\mathrm{TE}}}{\partial \phi}\right)\sin(k_p z) + \hat{\phi}\sum_{p=1}^{P}\left(\frac{jk_p\eta}{k\rho}\frac{\partial \psi_p^{\mathrm{TM}}}{\partial \phi} + \frac{\partial \psi_p^{\mathrm{TE}}}{\partial \rho}\right)\sin(k_p z) - \hat{z}\left(\frac{j\eta}{k}\right)\sum_{p=0}^{P}k_\rho^2\psi_p^{\mathrm{TM}}\cos(k_p z). \quad (38)$$

A Fourier series expansion of (31) is

$$\boldsymbol{E}_A = \hat{\phi}\sum_{p=1}^{P}E_{A\phi p}(\phi)\sin(k_p z) + \hat{z}\sum_{p=0}^{P}E_{Azp}(\phi)\cos(k_p z) \quad (39)$$

where $E_{A\phi p}(\phi)$ and $E_{Azp}(\phi)$ are given by (22) and (23) with l replaced by ϕ. Requiring that the ϕ- and z-components of (38) reduce to those of $\boldsymbol{E}_A$ of (39) on the lateral wall of the cavity where $\rho = a$, one obtains, using (32) and (33), the integral equations

$$-\frac{\eta k_\rho^2 a}{4k}\int_0^{2\pi} J_p(\phi')H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\,d\phi' = E_{Azp}(\phi) \quad (40)$$

$$\lim_{\rho\to a^-}\left\{\frac{a}{4j}\int_0^{2\pi} M_p(\phi')\frac{\partial H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|)}{\partial \rho}d\phi'\right\} = E_{A\phi p}(\phi) - \left(\frac{jk_p\eta}{ka}\right)\frac{\partial \psi_p^{\mathrm{TM}}(a,\phi)}{\partial \phi} \quad (41)$$

where $\boldsymbol{\rho}$ is the transverse radius vector to the point (a,ϕ) in (40) and (ρ,ϕ) in (41), and "$\rho \to a^-$" means that ρ approaches a from the interior.

Consider (40). Fourier series expansions of $J_p(\phi)$ and $E_{Azp}(\phi)$ are

$$J_p(\phi) = \sum_{n=0}^{N}\{A_{Jpn}\cos(n\phi) + B_{Jpn}\sin(n\phi)\} \quad (42)$$

$$E_{Azp}(\phi) = \sum_{n=0}^{N}\{A_{zpn}\cos(n\phi) + B_{zpn}\sin(n\phi)\} \quad (43)$$

where the B_{Jp0} and B_{zp0} terms are to be omitted. In view of the addition theorem for Hankel functions [9, (5–103)], the approximation

$$H_0^{(2)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|) \approx \sum_{n=0}^{N}\{\varepsilon_n H_n^{(2)}(k_\rho a)J_n(k_\rho\rho) \cdot \cos(n(\phi-\phi'))\}, \quad \rho < a \quad (44)$$

is introduced where N is a large integer and ε_n is given by (24). On the left-hand side of (44), $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ are the transverse radius vectors to the points (ρ,ϕ) and (a,ϕ'), respectively. Substituting (42), (43), and (44) with $\rho \to a^-$ into (40) and performing the integration, one finds that

$$\left(\frac{\pi a}{2j}\right)\sum_{n=0}^{N}\{\{A_{Jpn}\cos(n\phi) + B_{Jpn}\sin(n\phi)\} \times H_n^{(2)}(k_\rho a)J_n(k_\rho a)\} = \left(\frac{jk}{k_\rho^2\eta}\right)\sum_{n=0}^{N}\{A_{zpn}\cos(n\phi) + B_{zpn}\sin(n\phi)\}. \quad (45)$$

Substituting (42) and (44) into (32), one sees that $\psi_p^{\mathrm{TM}}(\rho,\phi)$ is the left-hand side of (45) with $J_n(k_\rho a)$ replaced by $J_n(k_\rho\rho)$. Equivalently, $\psi_p^{\mathrm{TM}}(\rho,\phi)$ is the left-hand side of (45) with the n^{th} term multiplied by $J_n(k_\rho\rho)/J_n(k_\rho a)$. Therefore, $\psi_p^{\mathrm{TM}}(\rho,\phi)$ is the right-hand side of (45) with the n^{th} term multiplied by $J_n(k_\rho\rho)/J_n(k_\rho a)$:

$$\psi_p^{\mathrm{TM}}(\rho,\phi) = \sum_{n=0}^{N}\{J_n(k_\rho\rho) \cdot \{A_{pn}^{\mathrm{TM}}\cos(n\phi) + B_{pn}^{\mathrm{TM}}\sin(n\phi)\}\} \quad (46)$$

where

$$A_{pn}^{\mathrm{TM}} = \left(\frac{jk}{k_\rho^2\eta J_n(k_\rho a)}\right)A_{zpn} \quad (47)$$

$$B_{pn}^{\mathrm{TM}} = \left(\frac{jk}{k_\rho^2\eta J_n(k_\rho a)}\right)B_{zpn}. \quad (48)$$

Consider (41). Fourier series expansions of $M_p(\phi)$ and $E_{A\phi p}(\phi)$ are

$$M_p(\phi) = \sum_{n=0}^{N}\{A_{Mpn}\cos(n\phi) + B_{Mpn}\sin(n\phi)\} \quad (49)$$

$$E_{A\phi p}(\phi) = \sum_{n=0}^{N} \{A_{\phi pn}\cos(n\phi) + B_{\phi pn}\sin(n\phi)\}. \quad (50)$$

Substituting (44), (46), (49), and (50) into (41), one obtains

$$\begin{aligned}\left(\frac{\pi a}{2j}\right)\sum_{n=0}^{N}\{\{A_{Mpn}\cos(n\phi) + B_{Mpn}\sin(n\phi)\}\\ \times H_n^{(2)}(k_\rho a)k_\rho J_n'(k_\rho a)\}\\ = \sum_{n=0}^{N}\left\{\left(A_{\phi pn} + \frac{nk_pB_{zpn}}{k_\rho^2 a}\right)\cos(n\phi)\right.\\ \left.+\left(B_{\phi pn} - \frac{nk_pA_{zpn}}{k_\rho^2 a}\right)\sin(n\phi)\right\} \quad (51)\end{aligned}$$

Substituting (49) and (44) into (33), one sees that $\psi_p^{\mathrm{TE}}(\rho,\phi)$ is the left-hand side of (51) with $k_\rho J_n'(k_\rho a)$ replaced by $J_n(k_\rho \rho)$. Equivalently, $\psi_p^{\mathrm{TE}}(\rho,\phi)$ is the left-hand side of (51) with the n^{th} term multiplied by $J_n(k_\rho\rho)/\{k_\rho J_n'(k_\rho a)\}$. Therefore, $\psi_p^{\mathrm{TE}}(\rho,\phi)$ is the right-hand side of (51) with the n^{th} term multiplied by $J_n(k_\rho\rho)/\{k_\rho J_n'(k_\rho a)\}$:

$$\begin{aligned}\psi_p^{\mathrm{TE}}(\rho,\phi) = \sum_{n=0}^{N}\{J_n(k_\rho\rho)\\ \cdot\{A_{pn}^{\mathrm{TE}}\cos(n\phi) + B_{pn}^{\mathrm{TE}}\sin(n\phi)\}\} \quad (52)\end{aligned}$$

where

$$A_{pn}^{\mathrm{TE}} = \left(\frac{1}{k_\rho J_n'(k_\rho a)}\right)\left\{A_{\phi pn} + \frac{nk_pB_{zpn}}{k_\rho^2 a}\right\} \quad (53)$$

$$B_{pn}^{\mathrm{TE}} = \left(\frac{1}{k_\rho J_n'(k_\rho a)}\right)\left\{B_{\phi pn} - \frac{nk_pA_{zpn}}{k_\rho^2 a}\right\}. \quad (54)$$

The results obtained in Section III can be concisely stated. Namely, the electromagnetic field $(\boldsymbol{E}(\boldsymbol{r}), \boldsymbol{H}(\boldsymbol{r}))$ in the circular cavity is given by (2) and (3) with $H_p^{\mathrm{TM}}(\boldsymbol{r})$, $E_p^{\mathrm{TM}}(\boldsymbol{r})$, $E_p^{\mathrm{TE}}(\boldsymbol{r})$, and $H_p^{\mathrm{TE}}(\boldsymbol{r})$ given in terms of ψ_p^{TM} and ψ_p^{TE} by (34)–(37) where ψ_p^{TM} and ψ_p^{TE} are given by (46) and (52).

IV. Direct Solution of the Helmholtz Equation

In this section, the quantities ψ_p^{TM} of (46)–(48) and ψ_p^{TE} of (52)–(54) will be verified by direct solution of the Helmholtz equation. In this solution, there is no explicit reference to any surface current density. The expansions (46) and (52) for ψ_p^{TM} and ψ_p^{TE} are, with the A_{pn}'s and the B_{pn}'s equal to unknown constants, valid from the onset because each of the functions $J_n(k_\rho\rho)\cos(n\phi)$ and $J_n(k_\rho\rho)\sin(n\phi)$ satisfies the Helmholtz equation and the space of their collection for $(n = 0, 1, 2, \ldots, N)$ and $(p = 0, 1, 2, \ldots, P)$ becomes complete as both N and P approach infinity. Note that although (46) and (52) are valid everywhere inside the circular cylindrical cavity, they are valid only for $\rho \le \rho_{\min}$ inside an arbitrary cylindrical cavity where $\rho_{\min}$ is the value of ρ at the boundary point closest to the z-axis. Therefore, the method described here, that of direct solution of the Helmholtz equation, is valid only for a circular cylindrical cavity.

In the method of direct solution of the Helmholtz equation, the A_{pn}'s and the B_{pn}'s in (46) and (52) are evaluated by requiring that the ϕ- and z-components of the electric field in the cavity reduce to those of $\boldsymbol{E}_A$ on the lateral wall ($\rho = a, 0 \le z \le L$). Using (39), (43), (50), and (38), one obtains

$$\begin{aligned}\left[\left(\frac{jk_p\eta}{k\rho}\right)\frac{\partial\psi_p^{\mathrm{TM}}}{\partial\phi} + \frac{\partial\psi_p^{\mathrm{TE}}}{\partial\rho}\right]_{\rho=a}\\ = \sum_{n=0}^{N}\{A_{\phi pn}\cos(n\phi) + B_{\phi pn}\sin(n\phi)\} \quad (55)\\ -\frac{jk_\rho^2\eta}{k}\left[\psi_p^{\mathrm{TM}}\right]_{\rho=a}\\ = \sum_{n=0}^{N}\{A_{zpn}\cos(n\phi) + B_{zpn}\sin(n\phi)\}. \quad (56)\end{aligned}$$

Substitution of (46) and (52) into (55) and (56) leads to (47), (48), (53), and (54), thus verifying ψ_p^{TM} of (46)–(48) and ψ_p^{TE} of (52)–(54).

V. The Modal Expansion Method

In this section, the modal expansion method is used to verify the EFIE solution presented in the last paragraph of Section III. The modal expansion method was used in [4] to find the electric and magnetic dyadic Green's functions of bounded regions. A detailed discription of that method is contained in [4] where the bounded region is a cavity excited by volume distributions of electric and magnetic currents $\boldsymbol{J}$ and $\boldsymbol{M}$ inside the cavity in addition to a specified tangential electric field on the boundary surface S of the cavity. Setting $\boldsymbol{J}$ and $\boldsymbol{M}$ to zero and replacing $\hat{n}\times\boldsymbol{E}(\boldsymbol{r}')$ by $\hat{n}\times\boldsymbol{E}_A(\boldsymbol{r}')$ in [4, Eqs. (21) and (23)], one obtains the following expressions for the electric and magnetic fields $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r})$ in the cavity:

$$\boldsymbol{E}(\boldsymbol{r}) = -\int_S \hat{n}\times\boldsymbol{E}_A(\boldsymbol{r}')\cdot\nabla'\times\overleftrightarrow{G}_e(\boldsymbol{r}'\,|\,\boldsymbol{r})\,ds' \quad (57)$$

$$\boldsymbol{H}(\boldsymbol{r}) = -j\omega\epsilon\int_S \hat{n}\times\boldsymbol{E}_A(\boldsymbol{r}')\cdot\overleftrightarrow{G}_m(\boldsymbol{r}'\,|\,\boldsymbol{r})\,ds' \quad (58)$$

where $\hat{n}$ is the unit normal vector that points outward from S. Also, $\nabla'\times\overleftrightarrow{G}_e$ and $\overleftrightarrow{G}_m$ are given by [4, (22) and (24)]. Substituting these $\nabla'\times\overleftrightarrow{G}_e$ and $\overleftrightarrow{G}_m$ and (39) into (57) and (58), using (50), (43), and [4, (26)–(28)], and finally interchanging m and n, one obtains

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) = \sum_{p,n}\left(-\frac{\hat{\rho}}{\rho}\frac{\partial}{\partial\phi} + \hat{\phi}\frac{\partial}{\partial\rho}\right)\\ \times\left\{(k_pa)\tilde{S}_{pn}'\frac{\partial C_{zpn}}{\partial\phi} + S_{pn}'C_{\phi pn}\right\}\\ \cdot a\sin(k_pz) - \sum_{p,n}\left(\hat{\rho}\frac{\partial}{\partial\rho} + \frac{\hat{\phi}}{\rho}\frac{\partial}{\partial\phi}\right)(\tilde{S}_{pn}C_{zpn})\\ \cdot k_pa^2\sin(k_pz) + \hat{z}\sum_{p,n}S_{pn}C_{zpn}\cos(k_pz) \quad (59)\end{aligned}$$

$$\boldsymbol{H}(\boldsymbol{r}) = \sum_{p,n} \left(\hat{\rho}\frac{\partial}{\partial \rho} + \frac{\hat{\phi}}{\rho}\frac{\partial}{\partial \phi}\right) \times \left\{(S'_{pn} - k^2a^2\tilde{S}'_{pn})\frac{\partial C_{zpn}}{\partial \phi} - k_p a S'_{pn} C_{\phi pn}\right\} \times \frac{j\cos(k_p z)}{k\eta} + \sum_{p,n}\left(\frac{\hat{\rho}}{\rho}\frac{\partial}{\partial \phi} - \hat{\phi}\frac{\partial}{\partial \rho}\right) \cdot (\tilde{S}_{pn}C_{zpn})\frac{jka^2\cos(k_p z)}{\eta} - \hat{z}\sum_{p,n}\left(k_p S'_{pn}\frac{\partial C_{zpn}}{\partial \phi} + k_\rho^2 a S'_{pn} C_{\phi pn}\right) \times \frac{j\sin(k_p z)}{k\eta} \tag{60}$$

where $n = 0, 1, 2, \cdots, N$ and $p = 0, 1, 2, \cdots, P$. Also, $C_{\phi pn}$ and C_{zpn} are the quantities in brackets on the right-hand sides of (50) and (43), respectively. Furthermore,

$$S_{pn} = 2\sum_{m=1}^{\infty} \frac{x_{nm} J_n(x_{nm}\rho/a)}{(x_{nm}^2 - k_\rho^2 a^2) J_{n+1}(x_{nm})} \tag{61}$$

$$\tilde{S}_{pn} = 2\sum_{m=1}^{\infty} \frac{J_n(x_{nm}\rho/a)}{x_{nm}(x_{nm}^2 - k_\rho^2 a^2) J_{n+1}(x_{nm})} \tag{62}$$

$$S'_{pn} = \left\{\begin{matrix} \frac{-2}{k_\rho^2 a^2}, & n = 0 \\ 0, & n = 1, 2, \ldots \end{matrix}\right\} + 2\sum_{m=1}^{\infty} \frac{x'^2_{nm} J_n(x'_{nm}\rho/a)}{(x'^2_{nm} - k_\rho^2 a^2)(x'^2_{nm} - n^2) J_n(x'_{nm})} \tag{63}$$

$$\tilde{S}'_{p0} = 0 \tag{64}$$

$$\tilde{S}'_{pn} = 2\sum_{m=1}^{\infty} \frac{J_n(x'_{nm}\rho/a)}{(x'^2_{nm} - k_\rho^2 a^2)(x'^2_{nm} - n^2) J_n(x'_{nm})}, \quad n = 1, 2, \ldots \tag{65}$$

where x_{nm} is the m^{th} positive root of J_n and x'_{nm} is the m^{th} positive root of J'_n. Equations (61)–(63) are valid for $n = 0, 1, 2, \ldots$.

Using [13, (54)–(57) on p. 72][1] and [14, Formulas 9.1.27], one obtains

$$S_{pn} = \frac{J_n(k_\rho \rho)}{J_n(k_\rho a)}, \qquad 0 \le \rho < a \tag{66}$$

$$\tilde{S}_{pn} = \frac{1}{k_\rho^2 a^2}\left(S_{pn} - \left(\frac{\rho}{a}\right)^n\right), \qquad 0 \le \rho < a \tag{67}$$

$$S'_{pn} = \frac{J_n(k_\rho \rho)}{k_\rho a J'_n(k_\rho a)}, \qquad 0 \le \rho \le a \tag{68}$$

$$\tilde{S}'_{pn} = \frac{1}{k_\rho^2 a^2}\left(S'_{pn} - \frac{1}{n}\left(\frac{\rho}{a}\right)^n\right), \qquad 0 \le \rho \le a. \tag{69}$$

Equations (66)–(68) are valid for $n = 0, 1, 2, \ldots$. However, (69) is valid only for $n = 1, 2, \ldots$. According to (64), $\tilde{S}'_{p0} = 0$. Substituting (64) and (66)–(69) into (59) and (60), one obtains (2) and (3) where $\boldsymbol{H}_p^{\text{TM}}(\boldsymbol{r})$, $\boldsymbol{E}_p^{\text{TM}}(\boldsymbol{r})$, $\boldsymbol{E}_p^{\text{TE}}(\boldsymbol{r})$, and $\boldsymbol{H}_p^{\text{TE}}(\boldsymbol{r})$ are given by (34)–(37) in which ψ_p^{TM} and ψ_p^{TE} are given by (46) and (52). Thus, the EFIE solution of Section III is verified.

For the circular cavity, the EFIE solution presented in the last paragraph of Section III is much simpler than the modal expansion solution (59) and (60) where quantities therein are defined in the rest of the paragraph containing (59) and (60). In the second from the last paragraph of Section V however, this modal expansion solution was reduced to the EFIE solution. This reduction was done by evaluating the summations with respect to m in the S's in (59) and (60). In [4], the Green's dyadic functions $\overleftrightarrow{G}_e(\boldsymbol{r}'|\boldsymbol{r})$ and $\overleftrightarrow{G}_m(\boldsymbol{r}'|\boldsymbol{r})$ therein were simplified to [4, (36), (40)] by evaluating the summations with respect to p (please correct [4, (36), (40)] by subtracting the $\hat{z}$ terms instead of adding them). Hence, the modal expansion (59) and (60) can be alternatively reduced by evaluating the summations with respect to p therein.

VI. Discussion

Consideration was given to the problem of determining the electromagnetic field in a source-free cavity excited by a known tangential electric field in an aperture in its wall. This problem can be solved by means of the modal expansion method described in [2, ch. 5], [3, ch. 3], and the references cited therein. Actually, the method described in [2] and [3] is most often called the Green's dyadic function method rather than the modal expansion method. However, since the Green's dyadic function in [2] and [3] is that inferred by the modal expansion, the method described in [2] and [3] is, in effect, the same as the modal expansion method.

To solve the problem cited in the previous paragraph, the EFIE method is, for several reasons, often more efficient than the modal expansion method. The actual field is source-free inside the cavity. In the EFIE method, the representation of the field is source-free inside the cavity. In the modal expansion method, the field inside the cavity is expressed as a sum of mode fields. Each mode field is source-free, but only at its resonant frequency. Hence, it is unlikely that any mode field is source-free at the operating frequency. As a result, the modal expansion is an expansion of a field that is source-free inside the cavity in terms of fields that are not source-free inside the cavity. Furthermore, the modal expansion for the electric field must converge to $\boldsymbol{E}_A$ immediately inside a hypothetical perfectly conducting wall that closes the aperture and must converge to zero exactly on this wall. Here, $\boldsymbol{E}_A$ is the aperture field and "inside" means on the side facing the interior of the cavity. Consequently, the modal expansion for the electric field must converge to a field that is discontinuous at all points of the aperture where $\boldsymbol{E}_A$ is not zero. Since each electric field in the modal expansion is continuous, the modal expansion for the electric field must converge nonuniformly and thus very slowly near points of the aperture where $\boldsymbol{E}_A$ is not zero. For an arbitrarily shaped cavity, the EFIE method is easier to implement than the modal expansion method. The EFIE method requires numerical solution of only one integral equation at the operating frequency whereas the modal expansion method requires determination of each frequency

[1] Strictly speaking, [13, (57)] is not correct because the contribution due to the trivial root of J'_0 was omitted; correction was necessary before use. See [15, Section 80].

at which the associated homogeneous integral equation has a nontrivial solution.

If the EFIE method is often more efficient than the modal expansion method, then why is the EFIE method not even mentioned in [2] and [3]? Four reasons can be given.

1) The expositions in [2] and [3] treat the more general problem where sources can be inside the cavity. The EFIE method is not directly applicable when such sources are present.
2) If knowledge of the resonant frequencies is desired, then solution by the modal expansion method is desirable because these frequencies appear explicitly in this solution. If solution is by the EFIE method, then the resonant frequencies must be computed as the frequencies at which this solution becomes infinite. Compare the modal expansion solution (59) and (60) where the roots x_{nm} and x'_{nm} appear explicitly (indicating that they must have been determined during the course of the solution) with the EFIE solution presented in the last paragraph of Section III where these roots do not appear explicitly.
3) The modal expansion solution is especially simple when the cavity is resonant or nearly resonant; in this case, the modal expansion, which is generally a triple summation [2, p. 383], is dominated by one term, namely the resonant mode.
4) The authors of [2] and [3] may have deemed the EFIE too simple to include in their expositions.

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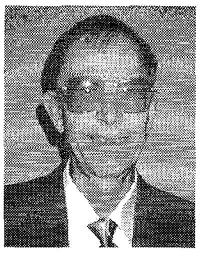

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